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Perturbation theory for the two-dimensional abelian Higgs model in the unitary gauge

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Abstract. In the unitary gauge the unphysical degrees of freedom of spontaneously broken gauge theories are eliminated. The Feynman rules are simpler than in other gauges, but it is non-renormalizable by the rules of power counting. On the other hand, it is formally equal to the limit $\xi \to 0$ of the renormalizable R_ξ-gauge. We consider perturbation theory to one-loop order in the R_ξ-gauge and in the unitary gauge for the case of the two-dimensional abelian Higgs model. An apparent conflict between the unitary gauge and the limit $\xi \to 0$ of the R_ξ-gauge is resolved, and it is demonstrated that results for physical quantities can be obtained in the unitary gauge.

1 Introduction

For theories of interacting gauge and Higgs fields with spontaneously broken gauge symmetry two well-known gauges are the unitary gauge (U-gauge) [1] and the renormalizable R_ξ-gauge [2,3]. In the U-gauge the gauge-variant transversal part of the Higgs field has been eliminated and the Lagrangian only contains physical degrees of freedom. Although the Feynman rules in the U-gauge are simpler, it is usually not used for perturbative calculations. The reason for this is the fact that for large momenta the gauge field propagator grows faster than in the R_ξ-gauge. Consequently the model in the U-gauge appears to be unrenormalizable by the usual power-counting rules. In the R_ξ-gauge, on the other hand, more fields have to be taken into account, namely the unphysical components of the Higgs field and the ghost field. The Feynman rules are more complicated and there are more diagrams to be calculated. The advantage is that the model in the R_ξ-gauge is manifestly renormalizable.

With the help of Slavnov–Taylor identities it can formally be shown that renormalized on-shell quantities are independent of the gauge [3–5]. Such physical quantities should therefore in principle be calculable in the U-gauge. It appears, however, that the cancellation of divergent terms is a delicate matter. In practice, calculations in the unitary gauge have sometimes led to results which are in conflict with those obtained in other gauges [6, 7].

In this paper we address the possibility of doing perturbation theory in the U-gauge and the relation between the R_ξ-gauge and the U-gauge. For simplicity we restrict ourselves to the two-dimensional abelian Higgs model. It contains all the features we would like to discuss, but the explicit calculations are easier than in non-abelian models in four dimensions. We perform the perturbative calculations of off-shell quantities on the one-loop level. Ultraviolet divergencies are treated by means of dimensional regularization, where the number of dimensions of space-time are taken to be $D = 2 - 2\epsilon$.

Formally, the U-gauge is obtained by taking the limit $\xi \to 0$ of the R_ξ-gauge. Applying this prescription naively, results are obtained which do not coincide with those of the U-gauge. We discuss the origin of this discrepancy, which is related to the fact that the limits $\epsilon \to 0$ and $\xi \to 0$ are not interchangeable, and discuss how to go from the R_ξ-gauge to the U-gauge properly.

We would like to point out that renormalization of the four-dimensional abelian Higgs model in the unitary gauge has been discussed by Sonoda [8] with the help of a suitable choice of interpolating fields.

2 The two-dimensional abelian Higgs model

The model contains a real vector field $A_\mu(x)$ and a complex scalar Higgs field $\phi(x)$. We shall consider the theory in a two-dimensional space-time with a Euclidean metric. The Lagrangian is

$$\mathcal{L} = \frac{1}{4} F_{\mu\nu} F_{\mu\nu} + |D_\mu \phi|^2 + V(\phi), \tag{1}$$

where

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \,, \tag{2}$$

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$$D_\mu = \partial_\mu - \mathrm{i}eA_\mu\,, \tag{3}$$

$$V(\phi) = -\frac{m^2}{2}|\phi|^2 + \frac{g}{6}|\phi|^4\,, \tag{4}$$

and e and g are coupling constants. The potential is of the mexican hat type with its minima at

$$|\phi| = \frac{v}{\sqrt{2}}\,, \quad \text{where} \quad v^2 = \frac{3m^2}{g}. \tag{5}$$

The Lagrangian is invariant under local gauge transformations

$$\phi(x) \to \phi'(x) = \mathrm{e}^{-\mathrm{i}\alpha(x)}\phi(x)\,, \tag{6}$$

$$A_\mu(x) \to A'_\mu(x) = A_\mu(x) - \frac{1}{e}\partial_\mu\alpha(x)\,. \tag{7}$$

2.1 Unitary gauge

The scalar field can be written in the form

$$\phi(x) = \rho(x)\,\mathrm{e}^{\mathrm{i}\omega(x)} \tag{8}$$

with real $\rho(x)$ and $\omega(x)$. The U-gauge is obtained by choosing the gauge transformation function as $\alpha(x) = \omega(x)$. The transformed fields are then

$$\phi'(x) = \rho(x), \tag{9}$$

$$A'_\mu(x) = A_\mu(x) - \frac{1}{e}\partial_\mu\omega(x) \doteq B_\mu(x), \tag{10}$$

$$F'_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu\,. \tag{11}$$

They represent the gauge invariant physical degrees of freedom. In terms of $\rho(x)$ the potential can be expressed as

$$V(\rho) = \frac{g}{6}\left(\rho^2 - \frac{v^2}{2}\right)^2 + \text{const.} \tag{12}$$

After expanding the scalar field around the minimum of the potential as

$$\rho(x) = \frac{1}{\sqrt{2}}(v + \sigma(x)), \tag{13}$$

the Lagrangian reads, up to an irrelevant constant,

$$\mathcal{L} = \frac{1}{4}F_{\mu\nu}F_{\mu\nu} + \frac{1}{2}e^2v^2B_\mu^2 + \frac{1}{2}(\partial_\mu\sigma)^2 + \frac{m^2}{2}\sigma^2 + \frac{gv}{3!}\sigma^3 + \frac{g}{4!}\sigma^4 + e^2v\sigma B_\mu^2 + \frac{1}{2}e^2\sigma^2B_\mu^2\,. \tag{14}$$

One can read off that the Higgs scalar σ has mass m and the vector field B_μ is massive with mass $m_v = ev$. From the Lagrangian the following Feynman rules are obtained.

- scalar propagator:

$$\Delta_\sigma(k) = (m^2 + k^2)^{-1}$$

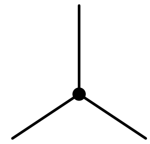
=

- gauge field propagator:

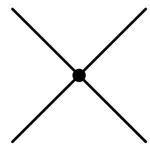
$$\Delta_{\mu\nu}(k) = (m_v^2 + k^2)^{-1}\left(\delta_{\mu\nu} + \frac{k_\mu k_\nu}{m_v^2}\right)$$

= μ ν

- σ^3-vertex: $-gv =$

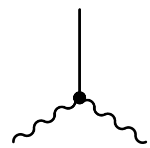


- σ^4-vertex: $-g =$



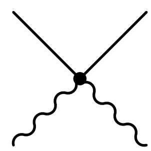
- σ-B_μ^2-vertex:

$-2e^2v\delta_{\mu\nu} =$



- σ^2-B_μ^2-vertex:

$-2e^2\delta_{\mu\nu} =$



With these Feynman rules one can write down expressions for various Green functions. In order not to overlook subtleties, it should be taken into account, however, that the functional integral measure for the scalar field $\sigma(x)$ is not the standard one. For each point x the measure is, up to a constant factor,

$$\mathrm{d}(\mathrm{Re}\,\phi(x))\,\mathrm{d}(\mathrm{Im}\,\phi(x)) = \rho(x)\mathrm{d}\rho(x)\mathrm{d}\omega(x)\,. \tag{15}$$

The functional integral measure for the scalar field is therefore

$$\prod_x (v + \sigma(x))\,\mathrm{d}\sigma(x) \doteq \det J \prod_x \mathrm{d}\sigma(x), \tag{16}$$

with [9]

$$J(x,y) = \delta(x-y)\,((v + \sigma(x)))\,. \tag{17}$$

One can try to argue that $\det J$ does not affect the perturbative results, at least in dimensional regularization [10]. But it is safer to keep this term for the moment. With the help of ghost fields we can write

$$\det J = \int \mathcal{D}c\mathcal{D}\bar{c}\;\mathrm{e}^{-S_{\mathrm{gh}}}\,, \tag{18}$$

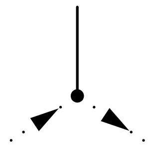
with

$$S_{\mathrm{gh}} = e^2v\int \mathrm{d}x\;\bar{c}(x)\,((v + \sigma(x)))\,c(x)\,. \tag{19}$$

The prefactor e^2v has been chosen such that comparison with similar terms in the R_ξ-gauge is facilitated. The Lagrangian gets the additional ghost terms

$$m_v^2\;\bar{c}c + e^2v\,\sigma\,\bar{c}c, \tag{20}$$

and the Feynman rules are augmented by

- ghost propagator: $(m_v^2)^{-1} =$ 
- ghost-σ-vertex: $-e^2 v =$ 

In the one-loop order the Green's functions of the scalar and vector fields get additional contributions from ghost loops. These are proportional to

$$\int \frac{\mathrm{d}^D k}{(2\pi)^D} \, \frac{1}{m_v^2} \,. \tag{21}$$

In dimensional regularization these contributions vanish due to the rule [11]

$$\int \frac{\mathrm{d}^D k}{(2\pi)^D} \, (k^2)^\alpha = 0 \,, \quad \text{for } \alpha \geq 0 \,. \tag{22}$$

This justifies one neglecting the measure factor det J. The ghost fields introduced above are, however, useful in the discussion of the relation between the U-gauge and the R_ξ-gauge.

2.2 R_ξ-gauge

In the R_ξ-gauge the Higgs field is decomposed into its real and imaginary parts as

$$\phi(x) = \frac{1}{\sqrt{2}} (v + \phi_1(x) + \mathrm{i}\phi_2(x)) \,. \tag{23}$$

Expanding the Lagrangian in terms of these fields, a mixing term between $A_\mu(x)$ and $\phi_2(x)$ appears on the quadratic level. The R_ξ-gauge is specified by the gauge fixing function

$$F = \partial_\mu A_\mu - \frac{ev}{\xi} \, \phi_2 \,, \tag{24}$$

where $\xi > 0$ is a real parameter. The gauge fixing term to be added to the Lagrangian is

$$\mathcal{L}_{\mathrm{gf}} = \frac{\xi}{2} F^2 \,. \tag{25}$$

It eliminates the $A_\mu(x)$–$\phi_2(x)$ mixing term. The gauge fixing procedure yields the Faddeev–Popov determinant det M_F, where

$$M_F = -\partial_\mu^2 + \frac{e^2 v}{\xi} (v + \phi_1) \,. \tag{26}$$

As usual it can be represented in terms of ghost fields via a ghost Lagrangian

$$\begin{aligned} \mathcal{L}_{\mathrm{gh}} &= \xi \, \bar{c}(x) M_F \, c(x) \qquad (27) \\ &= -\xi \, \bar{c}(x) \partial_\mu^2 c(x) + m_v^2 \, \bar{c}(x) c(x) + e^2 v \, \phi_1(x) \bar{c}(x) c(x) \,. \end{aligned}$$

By suitable normalization of the ghost fields the prefactor ξ has been chosen for later convenience. In contrast to the case of QED the ghost term cannot be neglected since it contains an interaction between the Higgs and the ghost fields.

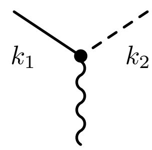
From the total Lagrangian the following Feynman rules are derived. In order to save space, the graphical representations are shown only for new types of propagators or vertices.

- ϕ_1 propagator: $\Delta_{\phi_1}(k) = (m^2 + k^2)^{-1}$
- ϕ_2 propagator:

$$\Delta_{\phi_2}(k) = \left(\frac{m_v^2}{\xi} + k^2 \right)^{-1} =$$

- gauge field propagator:

$$\Delta_{\mu\nu,\,\xi}(k) = (m_v^2 + k^2)^{-1} \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) + \frac{1}{\xi} \left(\frac{m_v^2}{\xi} + k^2 \right)^{-1} \frac{k_\mu k_\nu}{k^2}$$

- ghost propagator: $\Delta_c(k) = (m_v^2 + \xi k^2)^{-1}$
- ϕ_1^3-vertex: $-gv$
- ϕ_1^4-vertex: $-g$
- $\phi_1 \phi_2^2$-vertex: $-\frac{gv}{3}$
- $\phi_1^2 \phi_2^2$-vertex: $-\frac{g}{3}$
- ϕ_2^4-vertex: $-g$
- $A_\mu^2 \phi_1$-vertex: $-2e^2 v \delta_{\mu\nu}$
- $A_\mu \phi_1 \phi_2$-vertex: $-\mathrm{i}e(k_1 - k_2)_\mu =$ 

- $A_\mu^2 \phi_1^2$-vertex: $-2e^2 \delta_{\mu\nu}$
- $A_\mu^2 \phi_2^2$-vertex: $-2e^2 \delta_{\mu\nu}$
- $\phi_1 \bar{c} c$-vertex: $-e^2 v$

Comparing with the Feynman rules of the U-gauge we observe that in the limit $\xi \to 0$ the propagators and vertices involving the fields ϕ_1, A_μ and $\bar{c}, c$ go over to those of the fields σ, B_μ and $\bar{c}, c$ in the U-gauge. Moreover the ϕ_2 propagator

$$\Delta_{\phi_2}(k) = \frac{\xi}{m_v^2 + \xi k^2} \xrightarrow{\xi \to 0} 0 \tag{28}$$

vanishes in this limit. In this sense the U-gauge formally corresponds to the $\xi \to 0$ limit of the R_ξ-gauge [3, 10]. Indeed, in this limit the gauge fixing function forces the imaginary component ϕ_2 of the scalar field to vanish, which corresponds to the U-gauge.

Two other special cases are known in the literature. The limit $\xi \to \infty$ yields the Landau gauge, in which the vector propagator is purely transversal. The case $\xi = 1$ is the Feynman gauge, which has the simplest Feynman rules.

3 Perturbation theory

In this section we consider Green's functions of the abelian Higgs model in perturbation theory in one-loop order. For the treatment of divergencies we employ dimensional regularization with $D = 2 - 2\epsilon$ dimensions. The coupling constants are replaced by

$$\begin{aligned} e &\to \mu^{\epsilon} e\,, \\ g &\to \mu^{2\epsilon} g\,, \\ v &\to \mu^{-\epsilon} v\,, \end{aligned} \tag{29}$$

where μ is an arbitrary mass scale.

The one-loop corrections are of order g or e^2 relative to the tree level terms. As usual, fractions e^2/g are counted as being of order 1. Two-loop and higher corrections are of order g^2, ge^2 or e^4.

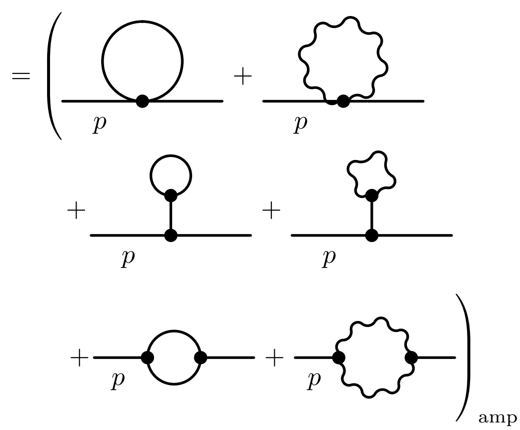
3.1 Scalar propagator

Let us start with the scalar propagator. We write its inverse as

$$G^{-1}(p) = m^2 + p^2 + \Sigma(p)\,, \tag{30}$$

where the self-energy $\Sigma(p)$ is given by the sum of one-particle irreducible, amputated propagator diagrams. For the σ propagator in the U-gauge we obtain

$-\Sigma_\sigma(p^2)$



$$\begin{aligned} = \frac{1}{4\pi} \Bigg\{ g \Bigg[&\left(1 - \frac{p^2}{3m^2}\right)\left(\frac{1}{\epsilon} - \gamma - \ln\frac{m^2}{4\pi\mu^2}\right) \\ &+ \frac{p^2}{3m^2} \ln\frac{3e^2}{g} \\ &+ \frac{3m^2}{2\sqrt{p^4 + 4p^2m^2}} \ln\frac{p^2 + 2m^2 + \sqrt{p^4+4p^2m^2}}{p^2 + 2m^2 - \sqrt{p^4+4p^2m^2}} \Bigg] \\ &+ e^2 \Bigg[2\left(\frac{1}{\epsilon} - \gamma - 2 - \ln\frac{m_v^2}{4\pi\mu^2}\right) \\ &+ \frac{(p^2+2m_v^2)^2}{2m_v^2\sqrt{p^4+4p^2m_v^2}} \ln\frac{p^2 + 2m_v^2 + \sqrt{p^4+4p^2m_v^2}}{p^2 + 2m_v^2 - \sqrt{p^4+4p^2m_v^2}} \Bigg] \Bigg\}\,, \end{aligned} \tag{31}$$

where $\gamma = 0.57721\ldots$ is Euler's constant.

From the propagator the renormalized mass and the wave function renormalization constant can be obtained. We shall consider two schemes here. In the first scheme the renormalized mass m_{R} and renormalization constant Z_{R} are defined by

$$G_\sigma^{-1}(p) = \frac{1}{Z_{\mathrm{R}}}\{m_{\mathrm{R}}^2 + p^2 + \mathcal{O}(p^4)\}, \tag{32}$$

which amounts to

$$Z_{\mathrm{R}}^{-1} = 1 + \left.\frac{\partial \Sigma_\sigma}{\partial p^2}\right|_{p=0} \tag{33}$$

$$m_{\mathrm{R}}^2 = Z_{\mathrm{R}}(m^2 + \Sigma_\sigma(0)). \tag{34}$$

This gives

$$\begin{aligned} m_{\mathrm{R}}^2 = m^2 &- \frac{g}{12\pi}\left[\frac{43}{12} + 4\left(\frac{1}{\epsilon} - \gamma - \ln\frac{m^2}{4\pi\mu^2}\right) - \ln\frac{3e^2}{g}\right] \\ &- \frac{e^2}{2\pi}\left[\frac{1}{\epsilon} - \gamma - 1 - \ln\frac{m_v^2}{4\pi\mu^2}\right], \end{aligned} \tag{35}$$

$$Z_{\mathrm{R}} = 1 + \frac{g}{4\pi m^2}\left[\frac{11}{36} - \frac{1}{3}\left(\frac{1}{\epsilon} - \gamma - \ln\frac{m_v^2}{4\pi\mu^2}\right)\right]. \tag{36}$$

In the second scheme the renormalized mass is taken to be the physical mass m_σ, given by the pole of the propagator,

$$G_\sigma^{-1}\left((\mathrm{i}m_\sigma, 0)\right) = 0\,, \tag{37}$$

and the wave function renormalization constant Z_σ is the corresponding residue,

$$G_\sigma(p) \simeq \frac{Z_\sigma}{p^2 + m_\sigma^2} \quad \text{for} \quad p^2 \to -m_\sigma^2\,. \tag{38}$$

We get

$$\begin{aligned} m_\sigma^2 = m^2 - \frac{g}{4\pi}\Bigg[&\frac{4}{3}\left(\frac{1}{\epsilon} - \gamma - \ln\frac{m^2}{4\pi\mu^2}\right) - \frac{1}{3}\ln\frac{3e^2}{g} \\ &+ 2\sqrt{3}\,\mathrm{arccot}\,\sqrt{3} \\ &+ \frac{2}{3}\frac{\left(1 - 6\frac{e^2}{g}\right)^2}{\sqrt{12\frac{e^2}{g} - 1}}\,\mathrm{arccot}\,\sqrt{12\frac{e^2}{g} - 1}\Bigg] \\ - \frac{e^2}{2\pi}&\left[\frac{1}{\epsilon} - \gamma - 2 - \ln\frac{m_v^2}{4\pi\mu^2}\right], \end{aligned} \tag{39}$$

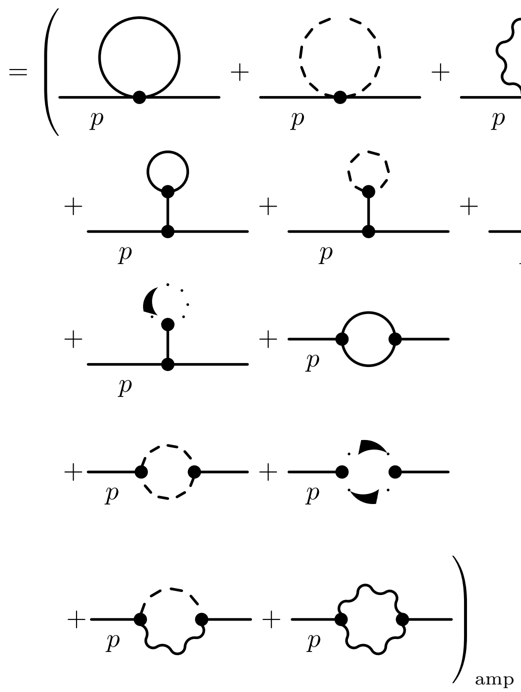
$$\begin{aligned} Z_\sigma = 1 + \frac{g}{4\pi m^2}\Bigg[&-1 + \frac{2\sqrt{3}}{3}\,\mathrm{arccot}\,\sqrt{3} \\ &- \frac{1}{3}\left(\frac{1}{\epsilon} - \gamma - \ln\frac{m_v^2}{4\pi\mu^2}\right) + \frac{\left(1 - 6\frac{e^2}{g}\right)^2}{3\left(1 - 12\frac{e^2}{g}\right)} \end{aligned} \tag{40}$$

$$+\frac{2}{3}\frac{\left(1-6\frac{e^2}{g}\right)\left(1-12\frac{e^2}{g}-36\frac{e^4}{g^2}\right)}{\left(12\frac{e^2}{g}-1\right)^{\frac{3}{2}}} \times \operatorname{arccot}\sqrt{12\frac{e^2}{g}-1}\Bigg],$$

where we assume $g \leq 12e^2$ for the analytic continuation.

The corresponding propagator in the R_ξ-gauge is the ϕ_1 propagator. Its self-energy in one-loop order is

$-\Sigma_{\phi_1,\mathrm{R}_\xi}(p^2)$



$$= \frac{1}{4\pi}\Bigg\{ g\Bigg[\frac{4}{3}\left(\frac{1}{\epsilon}-\gamma-\ln\frac{m^2}{4\pi\mu^2}\right)-\frac{1}{3}\ln\frac{3e^2}{g} + \frac{p^2+m^2}{3m^2}\ln\xi + \frac{3m^2}{2\sqrt{p^4+4m^2p^2}}\ln\frac{p^2+2m^2+\sqrt{p^4+4m^2p^2}}{p^2+2m^2-\sqrt{p^4+4m^2p^2}} + \frac{m^4-p^4}{6m^2\sqrt{p^4+4p^2\frac{m_v^2}{\xi}}}\ln\frac{p^2+2\frac{m_v^2}{\xi}+\sqrt{p^4+4p^2\frac{m_v^2}{\xi}}}{p^2+2\frac{m_v^2}{\xi}-\sqrt{p^4+4p^2\frac{m_v^2}{\xi}}}\Bigg] + e^2\Bigg[2\left(\frac{1}{\epsilon}-\gamma-2-\ln\frac{m_v^2}{4\pi\mu^2}\right) + \frac{(p^2+2m_v^2)^2}{2m_v^2\sqrt{p^4+4p^2m_v^2}}\ln\frac{p^2+2m_v^2+\sqrt{p^4+4p^2m_v^2}}{p^2+2m_v^2-\sqrt{p^4+4p^2m_v^2}}\Bigg]\Bigg\}. \quad (41)$$

This expression is valid if $p^2 \geq -4m^2$, $p^2 \geq -4m_v^2$ and $p^2 \geq -4m_v^2/\xi$ are fulfilled. For the renormalized masses and renormalization constants we obtain

$$m^2_{\mathrm{R,R}_\xi} = m^2 - \frac{g}{12\pi}\left[\frac{43}{12}+4\left(\frac{1}{\epsilon}-\gamma-\ln\frac{m^2}{4\pi\mu^2}\right) - \ln\frac{3e^2}{g} + \xi\frac{g}{6e^2} + \xi^2\frac{g^2}{108e^4}\right] - \frac{e^2}{2\pi}\left(\frac{1}{\epsilon}-\gamma-1-\ln\frac{m_v^2}{4\pi\mu^2}\right), \quad (42)$$

$$Z_{\mathrm{R},\xi} = 1+\frac{g}{4\pi m^2}\left[\frac{11}{36}+\frac{1}{3}\ln\xi-\xi^2\frac{g^2}{324e^4}\right] \quad (43)$$

in the first scheme, and

$$m^2_{\phi_1} = m^2 - \frac{g}{4\pi}\Bigg[\frac{4}{3}\left(\frac{1}{\epsilon}-\gamma-\ln\frac{m_v^2}{4\pi\mu^2}\right) - \frac{1}{3}\ln\frac{3e^2}{g} + 2\sqrt{3}\operatorname{arccot}\sqrt{3} + \frac{2}{3}\frac{\left(1-6\frac{e^2}{g}\right)^2}{\sqrt{12\frac{e^2}{g}-1}}\operatorname{arccot}\sqrt{12\frac{e^2}{g}-1}\Bigg] - \frac{e^2}{2\pi}\left[\left(\frac{1}{\epsilon}-\gamma-2-\ln\frac{m_v^2}{4\pi\mu^2}\right)\right], \quad (44)$$

$$Z_{\phi_1} = 1+\frac{g}{4\pi m^2}\Bigg[-1+\frac{2\sqrt{3}}{3}\operatorname{arccot}\sqrt{3} + \frac{1}{3}\ln\xi + \frac{\left(1-6\frac{e^2}{g}\right)^2}{3\left(1-12\frac{e^2}{g}\right)} + \frac{2}{3}\frac{\left(1-6\frac{e^2}{g}\right)\left(1-12\frac{e^2}{g}-36\frac{e^4}{g^2}\right)}{\left(12\frac{e^2}{g}-1\right)^{\frac{3}{2}}} \times\operatorname{arccot}\sqrt{12\frac{e^2}{g}-1} + \frac{4}{3\sqrt{12\frac{e^2}{\xi g}-1}}\operatorname{arccot}\sqrt{12\frac{e^2}{\xi g}-1}\Bigg] \quad (45)$$

in the second scheme.

In all cases the renormalized propagator

$$G_\mathrm{R}(p) = Z^{-1}\,G(p)\,, \quad (46)$$

expressed in terms of the renormalized mass, is finite. In the R_ξ-gauge it depends on the gauge parameter ξ. The renormalized mass $m_{\mathrm{R},\xi}$, not being a physical on-shell quantity, also depends on ξ. In contrast, the physical mass m_{ϕ_1} is independent of ξ as expected.

3.2 The U-gauge limit

Let us consider the relation between the two gauges. We should expect that the expressions calculated in the R_ξ-gauge go over to those in the U-gauge, if we let $\xi \to 0$. Indeed, for the masses we see that

$$\lim_{\xi \to 0} m_{\mathrm{R},\xi} = m_{\mathrm{R}} \tag{47}$$

in the first scheme, and

$$m_{\phi_1} = m_\sigma \tag{48}$$

in the second scheme.

For the renormalization constants, however, the situation is different. Both $Z_{\mathrm{R},\,\xi}$ and Z_{ϕ_1} contain a $(\log \xi)$-term and appear to diverge as $\xi \to 0$. Here the formal equivalence between the R_ξ-gauge in the limit $\xi \to 0$ and the U-gauge seems to break down.

Let us consider this discrepancy more carefully. The propagator gets contributions from ghost and ϕ_2-loops, which are of the form

$$I = \mu^{2\epsilon} \int \frac{\mathrm{d}^D k}{(2\pi)^D} \, \frac{1}{k^2 + \frac{m_v^2}{\xi}} \,. \tag{49}$$

In dimensional regularization this is

$$I = \frac{1}{4\pi} \left(\frac{m_v^2}{4\pi\mu^2\xi} \right)^{-\epsilon} \Gamma(\epsilon) \,. \tag{50}$$

If expanded for small ϵ in the usual way, it reads

$$I = \frac{1}{4\pi} \left(\frac{1}{\epsilon} - \gamma - \log \frac{m_v^2}{4\pi\mu^2} + \log \xi + \mathcal{O}(\epsilon) \right), \tag{51}$$

and we find the disturbing $(\log \xi)$-term. This expansion for small ϵ is, however, only applicable for fixed finite ξ. The ξ-dependence of I is contained in the factor

$$\frac{1}{\epsilon}\, \xi^\epsilon = \frac{1}{\epsilon} + \log \xi + \mathcal{O}(\epsilon) \,. \tag{52}$$

If the limit $\xi \to 0$ is taken first, with a positive ϵ, one gets instead

$$I \xrightarrow{\xi \to 0} 0 \,. \tag{53}$$

Alternatively, this can be obtained by writing

$$I = \xi \, \mu^{2\epsilon} \int \frac{\mathrm{d}^D k}{(2\pi)^D} \, \frac{1}{\xi k^2 + m_v^2} \tag{54}$$

and using the rule (22).

For the other terms involving gauge field loops the integrals are more complicated, but a detailed analysis shows that similar considerations hold.

We conclude that the limits $\epsilon \to 0$ and $\xi \to 0$ cannot be interchanged. As a consequence, the Laurent expansion in ϵ is not compatible with the limit $\xi \to 0$. In order to arrive at the U-gauge as a limit of the R_ξ-gauge, the limit has to be taken for fixed non-vanishing ϵ before the resulting expressions are expanded around $\epsilon = 0$.

In general the small ξ- and ϵ-dependence of a diagram in D dimensions is of the type $\xi^{\alpha(D_0 - D)}$. The number D of dimensions has then to be chosen sufficiently small, $D < D_0$, when taking the limit $\xi \to 0$. In the example above we have $\alpha = \frac{1}{2}$, $D_0 = 2$.

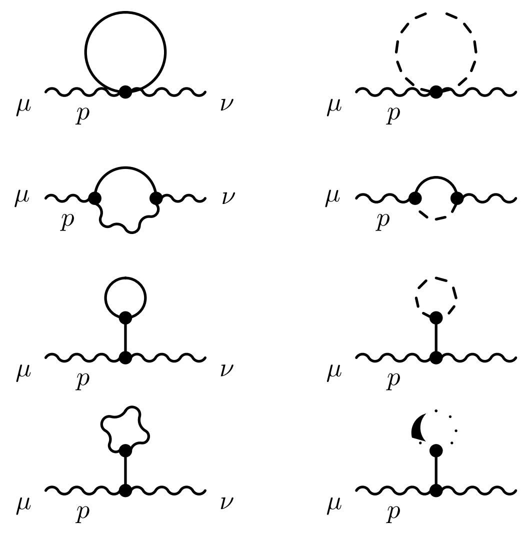
Taking these considerations into account, the limit $\xi \to 0$ can be taken for the self-energy, and the resulting expression coincides with the one in the U-gauge. Consequently the renormalized masses and renormalization constants also coincide in this limit.

3.3 Gauge field propagator

The inverse gauge field propagator can be decomposed into a transversal and a longitudinal part:

$$\begin{aligned} G^{-1}_{\mu\nu}(p) = &\left[\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right] \left[m_v^2 + p^2 + \Pi_1(p^2) \right] \\ &+ \frac{p_\mu p_\nu}{p^2} \left[m_v^2 + \xi p^2 + \Pi_2(p^2) \right] . \end{aligned} \tag{55}$$

In the R_ξ-gauge the diagrams



(56)

yield

$$\begin{aligned} &\Pi_1(p^2) \\ &= -\frac{e^2}{4\pi} \left[4 \left(\frac{1}{\epsilon} - \gamma + 1 - \ln \frac{m^2}{4\pi\mu^2} \right) \right. \\ &\quad - \frac{p^2 + m^2 - m_v^2}{p^2} \ln \frac{3e^2}{g} \\ &\quad - \frac{(p^2 + m^2 - m_v^2)^2}{p^2 \sqrt{(p^2 + m^2 - m_v^2)^2 + 4p^2 m_v^2}} \end{aligned}$$

$$\times \ln \frac{p^2+m^2+m_v^2+\sqrt{(p^2+m^2-m_v^2)^2+4p^2m_v^2}}{p^2+m^2+m_v^2-\sqrt{(p^2+m^2-m_v^2)^2+4p^2m_v^2}}$$
$$+\frac{6e^2}{g}\left(\frac{1}{\epsilon}-\gamma-2-\ln\frac{m_v^2}{4\pi\mu^2}\right)\Bigg], \tag{57}$$

$$\Pi_{2,\xi}(p^2)$$
$$= -\frac{e^2}{4\pi}\Bigg[4\left(\frac{1}{\epsilon}-\gamma-\ln\frac{m^2}{4\pi\mu^2}\right)$$
$$+\frac{6e^2}{g}\left(\frac{1}{\epsilon}-\gamma-2-\ln\frac{m_v^2}{4\pi\mu^2}\right)+2\ln\xi$$
$$-\frac{p^2-m^2+m_v^2}{p^2}\ln\frac{3e^2}{g}$$
$$+\frac{\sqrt{(p^2+m^2-m_v^2)^2+4p^2m_v^2}}{p^2}$$
$$\times \ln \frac{p^2+m^2+m_v^2+\sqrt{(p^2+m^2-m_v^2)^2+4p^2m_v^2}}{p^2+m^2+m_v^2-\sqrt{(p^2+m^2-m_v^2)^2+4p^2m_v^2}}$$
$$-\frac{p^2+2m^2-2\frac{m_v^2}{\xi}}{\sqrt{\left(p^2+m^2-\frac{m_v^2}{\xi}\right)^2+4p^2\frac{m_v^2}{\xi}}} \tag{58}$$
$$\times \ln \frac{p^2+m^2+\frac{m_v^2}{\xi}+\sqrt{\left(p^2+m^2-\frac{m_v^2}{\xi}\right)^2+4p^2\frac{m_v^2}{\xi}}}{p^2+m^2+\frac{m_v^2}{\xi}-\sqrt{\left(p^2+m^2-\frac{m_v^2}{\xi}\right)^2+4p^2\frac{m_v^2}{\xi}}}\Bigg].$$

The transversal part is manifestly independent of the gauge parameter ξ and is identical to the one in the U-gauge. This is generally true, as has been discussed in [12, 13], to all orders in perturbation theory. The renormalized vector mass and corresponding renormalization factor are derived from the transversal propagator and are equal, too, in both gauges. One obtains

$$m_{\mathrm{R},v}^2 = m_v^2 - \frac{e^2}{2\pi}\left[2\left(\frac{1}{\epsilon}-\gamma-\ln\frac{m^2}{4\pi\mu^2}\right)+1\right]$$
$$-\frac{3e^4}{4\pi g}\Bigg[2\left(\frac{1}{\epsilon}-\gamma-2-\ln\frac{m_v^2}{4\pi\mu^2}\right)$$
$$+\frac{1}{\left(1-\frac{3e^2}{g}\right)^2}-\frac{2}{\left(1-\frac{3e^2}{g}\right)^3}\ln\frac{3e^2}{g}\Bigg]$$
$$-\frac{9e^6}{4\pi g^2}\left[-\frac{7}{\left(1-\frac{3e^2}{g}\right)^2}+\frac{2}{\left(1-\frac{3e^2}{g}\right)^3}\ln\frac{3e^2}{g}\right]$$
$$+\frac{27e^8}{4\pi g^3}\frac{6}{\left(1-\frac{3e^2}{g}\right)^3}\ln\frac{3e^2}{g}, \tag{59}$$

$$Z_{\mathrm{R},v} = 1 \tag{60}$$
$$+\frac{e^2}{4\pi m_v^2}\left[\frac{7m_v^4-m^2m_v^2}{(m^2-m_v^2)^2}+2m_v^4\frac{m^2+2m_v^2}{(m^2-m_v^2)^3}\ln\frac{3e^2}{g}\right]$$

in the first renormalization scheme, and for the pole mass

$$m_A^2 = m_v^2$$
$$-\frac{g}{12\pi}\left[\ln\frac{3e^2}{g}+\frac{2\left(1-6\frac{e^2}{g}\right)^2}{\sqrt{12\frac{e^2}{g}-1}}\arctan\sqrt{12\frac{e^2}{g}-1}\right]$$
$$-\frac{e^2}{2\pi}\left[2\left(\frac{1}{\epsilon}-\gamma+1-\ln\frac{m^2}{4\pi\mu^2}\right)-\ln\frac{3e^2}{g}\right]$$
$$-\frac{3e^4}{2\pi g}\left[\frac{1}{\epsilon}-\gamma-2-\ln\frac{m_v^2}{4\pi\mu^2}\right], \tag{61}$$

$$Z_v = 1$$
$$+\frac{g}{12\pi m_v^2}\Bigg[\ln\frac{3e^2}{g}-2\frac{1-21\frac{e^2}{g}+108\frac{e^4}{g^2}-108\frac{e^6}{g^3}}{\left(12\frac{e^2}{g}-1\right)^{\frac{3}{2}}}$$
$$\times\arctan\sqrt{12\frac{e^2}{g}-1}\Bigg]$$
$$+\frac{e^2}{4\pi m_v^2}\left[-\ln\frac{3e^2}{g}-2\frac{\left(1-6\frac{e^2}{g}\right)^2}{12\frac{e^2}{g}-1}\right] \tag{62}$$

in the second scheme, where again we assume $g \leq 12e^2$.

Using the correct prescription for taking the limit $\xi \to 0$, one finds that the longitudinal part $\Pi_{2,\xi}(p^2)$ goes over to the result of the U-gauge.

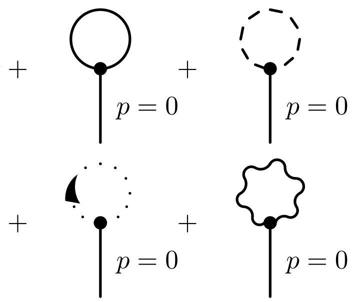
In one-loop order the mixing between the A_μ and ϕ_2 reappears. We do not display our results for the ϕ_2 propagator and the A_μ–ϕ_2 mixing, because the Slavnov–Taylor identities guarantee that contributions from the longitudinal gauge field, the ϕ_2 field and the ghosts cancel in physical amplitudes [4, 5].

We also calculated the ghost propagator in the R_ξ-gauge, but do not display the result here. It develops a pole at a non-vanishing ghost mass. In the limit $\xi \to 0$ the ghost mass goes to infinity, as it should [3,5], and the ghost propagator goes over into the static one of the U-gauge.

3.4 Field expectation value

The vacuum expectation value of the Higgs field gets contributions from one-loop diagrams. In the R_ξ-gauge one gets

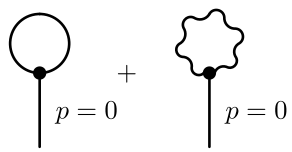
$v_{\mathrm{R}_\xi} = v +$



$$= \mu^{-\epsilon} v \left\{ 1 + \frac{g}{4\pi m^2} \left[-\frac{2}{3} \left(\frac{1}{\epsilon} - \gamma - \ln \frac{m^2}{4\pi\mu^2} \right) + \frac{1}{6} \ln \frac{3e^2}{g} - \frac{1}{6} \ln \xi \right] - \frac{e^2}{4\pi m^2} \left(\frac{1}{\epsilon} - \gamma - 2 - \ln \frac{m_v^2}{4\pi\mu^2} \right) \right\}. \quad (63)$$

In the proper limit it approaches the result of the U-gauge

$v_U = v +$



$$= \mu^{-\epsilon} v \left\{ 1 - \frac{g}{8\pi m^2} \left[\frac{1}{\epsilon} - \gamma - \ln \frac{m^2}{4\pi\mu^2} \right] - \frac{e^2}{4\pi m^2} \left[\frac{1}{\epsilon} - \gamma - 2 - \ln \frac{m_v^2}{4\pi\mu^2} \right] \right\}. \quad (64)$$

The expressions for the field expectation value can be renormalized by multiplication with an appropriate scalar field renormalization factor $Z^{-1/2}$ and expressing the bare couplings and masses by their renormalized counterparts. This would require the calculation of three-point vertices in the one-loop approximation.

The vacuum expectation value of the scalar field is not independent of the gauge parameter ξ, even after it is renormalized. This is not unexpected [6, 10, 14, 15], since it is an off-shell quantity.

4 Conclusion

The two-dimensional abelian Higgs model has been studied in the R_ξ-gauge and in the unitary gauge in the framework of dimensional regularization, where $D = 2 - 2\epsilon$. The propagators and field expectation values have been calculated on the one-loop level. An apparent discrepancy between the two gauges has been resolved, and it has been shown that the results in the unitary gauge can be obtained from those of the R_ξ-gauge by taking the limit $\xi \to 0$ before removing the dimensional regularization via $\epsilon \to 0$. The resulting renormalized propagators are finite off-shell. The unitary gauge appears to be suitable for the calculation of physical quantities.

It is, however, not possible to obtain the results for off-shell (ξ-dependent) quantities in the unitary gauge by taking the limit $\xi \to 0$ of the final renormalized results in the R_ξ-gauge after the regularization has been removed. For physical ξ-independent quantities this reservation does not apply.

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